

**2022 FALL REAL ANALYSIS (I) @ NCTU APPL. MATH.
HOMEWORK 1**

- Please answer the following questions in details, which means you need to state all theorems and all reasons you have been using.
 - Please mark your name, student ID, and question numbers clearly on your answer sheet. The deadline to hand in the exercise is on September 23, 2022.
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- (1) Let $R \subset \mathbb{R}^n$ be a *rectangle*, prove that $m_*(R) = |R|$.
- (2) Let $E \subset \mathbb{R}$ be a set, the outer Jordan content is defined by

$$J_*(E) := \inf \left\{ \sum_{j=1}^J |Q_j| : E \subset \cup_{j=1}^J Q_j \right\}$$

where Q_j 's are intervals in \mathbb{R} for $j = 1, 2, \dots, J$.

- (a) Prove that $J_*(E) = J_*(\overline{E})$, where \overline{E} denotes the closure of E .
- (b) Construct an example such that $J_*(E) = 1$ but $m_*(E) = 0$, where $m_*(E)$ is the outer measure of E .
- (3) Let $E_1, E_2 \subset \mathbb{R}$ be measurable sets. Show that $E_1 \times E_2$ is a measurable set in \mathbb{R}^2 and $m(E_1 \times E_2) = m(E_1) \cdot m(E_2)$ ¹.

¹In this case we *define* $0 \cdot \infty = 0$